KENDRIYA VIDYALAYA SANGATHAN, CHENNAI REGION

CLASS-XII-REVISION EXAMINATION-2012-2013-MATHEMATICS

SCORING KEY

1	$\lvert 3AB \rvert = 27\lvert A\rvert\lvert B\rvert = -54$	1
2	$B^2 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} i^2 & 0 \\ 0 & i^2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$ $B^4 = I^2 = I$ $B^{4n} = I, n \in N$	1
3	$\int e^x 7^x\, dx = \int (7e)^x\, dx = \frac{(7e)^x}{\log(7e)} + C$	1
4	$\cos^{-1}\left(\frac{a}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ $= \cos^{-1}\left(\frac{a}{5}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$ $\Rightarrow a = 3$	1
5	$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\, dx -----(1)$ $I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\, dx -----(2)$ $(1)+(2) \Rightarrow 2I = \int_1^3 \left[\frac{\sqrt{4-x}+\sqrt{x}}{\sqrt{4-x}+\sqrt{x}}\right] dx \Rightarrow 2I = \int_1^3 dx \Rightarrow 2I = [x]_1^3 = 2 \Rightarrow I = 1$	1
6	Condition for coplanarity $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$ $\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ p & p-2 & -1 \end{vmatrix} = 0$ $\Rightarrow 1[1-2(p-2] - 1[-1-2p] + 1[p-2+p] = 0$ $\Rightarrow 5-2p+1+2p+2p-2 = 0$ $\rightarrow 2p+4 = 0 \Rightarrow p = -2$	1

7	If $\begin{pmatrix} 0 & x+3y \\ 12 & x \end{pmatrix}$ is a skew symmetric matrix then x= 0 (diagonal elements of a skew symmetric matrix is 0) and $a_{ij} = -a_{ji} \Rightarrow a_{12} = -a_{21} \Rightarrow x+3y = -12 \Rightarrow y = -4$ (or) $A^T = -A$ for a skew symmetric matrix	1
8	$A(1,2,-3),\ B(-1,-2,1)$ $\vec{AB} = (-2,-4,4)$ $\left\vert\vec{AB}\right\vert = \sqrt{(-2)^2+(-4)^2+(4)^2} = \sqrt{36} = 6$ direction cosines of $\vec{AB} = \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right) = \left(\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$	1
9	$\vert\vec{a}\vert = \vert\vec{b}\vert, \vartheta = 60^\circ$ $\vec{a}.\vec{b} = \vert\vec{a}\vert\vert\vec{b}\vert\cos\vartheta = 8$ $\Rightarrow \vert\vec{a}\vert^2 \times \frac{1}{2} = 8$ $\Rightarrow \vert\vec{a}\vert^2 = 16$ $\Rightarrow \vert\vec{a}\vert = 4 = \vert\vec{b}\vert$	1
10	R= $\{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3)(3,1)\}$ is the only relation which is reflexive , symmetric but not transitive .	1
11	$y = \log\left[x+\sqrt{x^2+a^2}\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left[1+\frac{2x}{2\sqrt{x^2+a^2}}\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left[1+\frac{x}{\sqrt{x^2+a^2}}\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left[\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right]$ $\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$ $\Rightarrow (x^2+a^2)\left(\frac{dy}{dx}\right)^2 = 1$ $\Rightarrow (x^2+a^2)2\frac{dy}{dx}.\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 = 0$ $\Rightarrow (x^2+a^2)\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right) = 0$	1 1 ½ 1 ½

11	(OR) substitute $x^2 = \cos 2\vartheta$ $u = \tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ $= \tan^{-1}\sqrt{\frac{1-\cos 2\vartheta}{1+\cos 2\vartheta}} = \tan^{-1}\sqrt{\frac{2\sin^2\vartheta}{2\cos^2\vartheta}} = \tan^{-1}(\tan\vartheta) = \vartheta$ $= \frac{1}{2}\cos^{-1}x^2 = \frac{1}{2}v \quad where \ v = \cos^{-1}x^2$ $\therefore \quad u = \frac{1}{2}v$ $\frac{du}{dv} = \frac{1}{2}$	½ 1½ 1 1
12	From 0 to 9 the prime numbers are 2 , 3 , 5 , 7 X = 0 , 1,2 p(x = 0) = $\frac{6}{10}\times\frac{5}{9} = \frac{30}{90} = \frac{10}{30}$ P(x=1)= $2\left(\frac{4}{10}\times\frac{6}{9}\right) = \frac{48}{90} = \frac{16}{30}$ p(x=3)= $\frac{4}{10}\times\frac{3}{9} = \frac{12}{90} = \frac{4}{30}$ X: 0, 1, 2 — p(x): $\frac{10}{30}$, $\frac{16}{30}$, $\frac{4}{30}$ Mean = $\sum xp(x) = 0 + 1\times\frac{16}{30} + 2\times\frac{4}{30} = \frac{24}{30} = \frac{4}{5}$ $E(x^2) = \sum x^2 P(x) = 0 + 1\times\frac{16}{30} + 4\times\frac{4}{30} = \frac{32}{30} = \frac{16}{15}$ $\text{var}(x) = E(x^2) - [E(x)]^2 = \frac{32}{30} - \left(\frac{4}{5}\right)^2 = \frac{16}{15} - \frac{16}{25} = \frac{32}{75}$	1 1 1 1
13	let the adjacent sides of the parallelogram be $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ area of parallelogram = $\lvert\vec{a}\times\vec{b}\rvert$ $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 4 & -5\\ 1 & 2 & 3\end{vmatrix} = 22\hat{i} - 11\hat{j} \Rightarrow \lvert\vec{a}\times\vec{b}\rvert = 11\sqrt{5}$ vectors parallel to diagonals are $\vec{a} + \vec{b} \ and \ \vec{a} - \vec{b}$	2

X	0	1	2
p(x)	$\frac{10}{30}$	$\frac{16}{30}$	$\frac{4}{30}$

	Let $\vec{a}+\vec{b}=\vec{c}$ $\Rightarrow \vec{c}=3\hat{i}+6\hat{j}-2\hat{k}$, $\lvert\vec{c}\rvert=\sqrt{49}=7$ unit vector parallel to $\vec{c}=\frac{\vec{c}}{\lvert\vec{c}\rvert}=\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ **Let** $\vec{a}-\vec{b}=\vec{d}$ $\vec{d}=\hat{i}+2\hat{j}-8\hat{k},\lvert\vec{d}\rvert=\sqrt{69}$ unit vector parallel to $\vec{d}=\frac{\vec{d}}{\lvert\vec{d}\rvert}=\frac{1}{69}\left(\hat{i}+2\hat{j}-8\hat{k}\right)$	1 1
14	$2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ $\frac{dx}{dy}=-\frac{y-2xe^{\frac{x}{y}}}{2ye^{\frac{x}{y}}}$ $\frac{dx}{dy}=-\frac{1}{2e^{\frac{x}{y}}}+\frac{x}{y}$ substitute x= vy $\frac{dx}{dy}=v+y\frac{dv}{dy}$ $v+y\frac{dv}{dy}=-\frac{1}{2v}+v\Rightarrow y\frac{dv}{dy}=-\frac{1}{2v}\Rightarrow\int 2v\,dv=-\int\frac{dy}{y}$ $\Rightarrow 2\frac{v^2}{2}=-\log y+c\Rightarrow v^2=-\log y+c\Rightarrow\frac{x^2}{y^2}=-\log y+c$ x= 0 , y = 1 $\Rightarrow$ 0 = $-$ log 1 + c $\Rightarrow c=0$ hence $\frac{x^2}{y^2}=-\log y$ **(OR)** $\log\left(\frac{dy}{dx}\right)=3x+4y$ $\Rightarrow\frac{dy}{dx}=e^{3x+4y}$ $\Rightarrow\frac{dy}{dx}=e^{3x}.e^{4y}$	½ 1 ½ 1 ½ ½ 1 ½

	$\Rightarrow \int \frac{dy}{e^{4y}} = \int e^{3x} dx$	½
	$\Rightarrow \int e^{-4y} dy = \int e^{3x} dx$	
	$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$	1
	x = 0 , y = 0	
	$\Rightarrow -\frac{1}{4} = \frac{1}{3} + c \Rightarrow c = -\frac{1}{4} - \frac{1}{3} = -\frac{7}{12}$	½
	hence	
	$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$	½
15	$(a,b)*(c,d) = (a+c, b+d) \Rightarrow (c+a, d+b) \Rightarrow (c,d)*(a,b)$	1
	hence * is commutative	
	$[(a,b)*(c,d)]*(e,f) = (a+c, b+d)*(e,f) = (a+c+e, b+d+f)$	
	$= [a+(c+e), b+(d+f)]$	
	$= (a,b)*[(c+e, d+f)] = (a,b)*[(c,d)*(e,f)]$	2
	hence * is associative	
	let (c, d) be the identity element of (a, b) then (a, b) * (c, d) = (a, b)	
	$\Rightarrow (a+c, b+d) = (a,b) \Rightarrow a+c = a.,\ b+d = b \Rightarrow a = 0, b = 0 \notin N$	1
	identity element for A on * does not exist	
	(OR)	1
	For any $a \in A$, $\lvert a-a \rvert = 0$ is even $\Rightarrow (a,a) \in R$,hence R is reflexive	
	$(a,b) \in R \Rightarrow \lvert a-b \rvert$ is even $\Rightarrow \lvert b-a \rvert$ is even $\Rightarrow (b.a) \in R$	1
	hence R is symmetric	
	$(a,b) \in R \Rightarrow \lvert a-b \rvert$ is even $\Rightarrow$ a – b is even	
	$(b,c) \in R \Rightarrow \lvert b-c \rvert$ $\Rightarrow$ b-c is even	1½
	a- b + b-c is even $\Rightarrow$ a-c is even $\Rightarrow (a,c) \in R$	
	hence R is transitive	½
	since R is reflexive , symmetric and transitive it is an equivalence relation	
16	$\int \frac{5x-2}{3x^2+2x+1} dx$	
	5x-2 =A $\frac{d}{dx}$ (1+2x+3x^2) +B = A(2+ 6x) + B	
	A= $\frac{5}{6}$,B= $-\frac{11}{3}$	1
	$I = \frac{5}{6} \int \frac{2+6x}{3x^2+2x+1} dx - \frac{11}{3} \int \frac{1}{3x^2+2x+1} dx$	1

	$I = \frac{5}{6} I_1 - \frac{11}{3} I_2$ $I_1 = \log\lvert 3x^2 + 2x + 1\rvert$ $I_2 = \int \frac{dx}{3x^2+2x+1} == \frac{1}{3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}} = \frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}$	½
	$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2} = \frac{1}{3}\times\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$	1
	I = 5/6 log $\lvert 3x^2 + 2x + 1\rvert$- $\frac{11}{3\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}}$ +c	½
	(OR)	
	$\int \frac{dx}{x(x^5+1)} = \int \frac{x^4}{x^5(x^5+1)}dx$ *substitute* $x^5 = t$	½
	$5x^4dx = dt \Rightarrow x^4dx = \frac{1}{5}dt$	½
	$I = \frac{1}{5}\int \frac{dt}{t(t+1)}$	½
	$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$ $1 = A(t+1) + Bt$	1
	$A = 1, B = -1$	
	$I = \frac{1}{5}\left[\int \frac{dt}{t} - \int \frac{dt}{t+1}\right] = \frac{1}{5}\left[\log t - \log(t+1)\right] = \frac{1}{5}\left[\frac{\log t}{\log(t+1)}\right] = \frac{1}{5}\left[\log \frac{x^5}{(x^5+1)}\right] + C$	1½
17	$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$, $y = \cos t \Rightarrow \frac{dy}{dt} = -\sin t \Rightarrow \frac{dy}{dx} = -\frac{\sin t}{\cos t} = -\tan t$	1
	$\frac{dy}{dx} = -1 \ at \ t = \frac{\pi}{4}$	1
	$x_1 = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad y_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	1
	equation of tangent $y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right) \Rightarrow \sqrt{2}x + \sqrt{2}y = 2$	
	equation of normal $y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right) \Rightarrow \sqrt{2}x - \sqrt{2}y = 0$	1

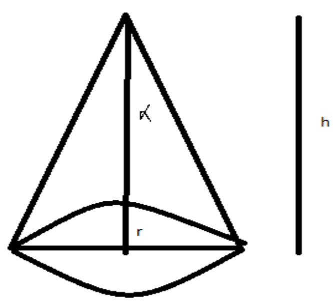
18	$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ $= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$ $= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$ $= (1+\text{xyz}) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$ $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$ $= (1+\text{xyz}) \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} = (1+\text{xyz})(\text{x-y})(\text{y-z}) \begin{vmatrix} 0 & 1 & \text{x+y} \\ 0 & 1 & \text{y+z} \\ 1 & \text{z} & \text{z}^2 \end{vmatrix}$ $= (\text{x - y})(\text{y- z})(\text{z- x})(1+\text{xyz}) = 0$ since $x \neq y \neq z \Rightarrow 1+xyz = 0 \Rightarrow xyz = -1$	½ 1 1 1 ½
19	$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$ $\tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$ $= \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right) = \tan^{-1}\left[\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \tan\frac{x}{2}\right)\right] = \frac{\pi}{4} - \frac{x}{2}$	1+1 4(½)
20	$\underset{x\to 0^+}{lt} f(x) = \underset{x\to 0-}{lt} f(x) = f(0)$ $\underset{x\to 0^+}{lt} f(x) = \underset{x\to 0^+}{lt} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} = \underset{x\to 0^+}{lt} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} \times \frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4}$ $= \underset{x\to 0^+}{lt} \frac{\sqrt{x}}{16+\sqrt{x}-16} \times \sqrt{16+\sqrt{x}}+4 = \underset{x\to 0^+}{lt} \frac{\sqrt{x}}{\sqrt{x}} \times \underset{x\to 0^+}{lt} \sqrt{16+\sqrt{x}}+4 = 8$	1½

<table>
<tr><td></td><td>$\underset{x\to 0-}{lt} f(x) = \underset{x\to 0-}{lt} \frac{1-\cos 4x}{x^2} = \underset{x\to 0-}{lt} \frac{2\sin^2 2x}{x^2} = \underset{x\to 0-}{lt} \frac{2\sin^2 2x}{4x^2} \times 4 = 8 \underset{2x\to 0-}{lt} \left(\frac{\sin 2x}{2x}\right)^2 = 8$
$\therefore f(0) = a = 8$

Thoughts affecting the continuity while writing exam- ant two factors
Lack of preparation, movement of students in the corridor, friend asking for answer, announcement of teacher regarding time, noise of the bell , any other distractions. The thoughts affecting continuity while writing exam are unavoidable at many occasions. We should learn and practice to avoid external disturbances and concentrate on our work to reach our goal.</td><td>1
½

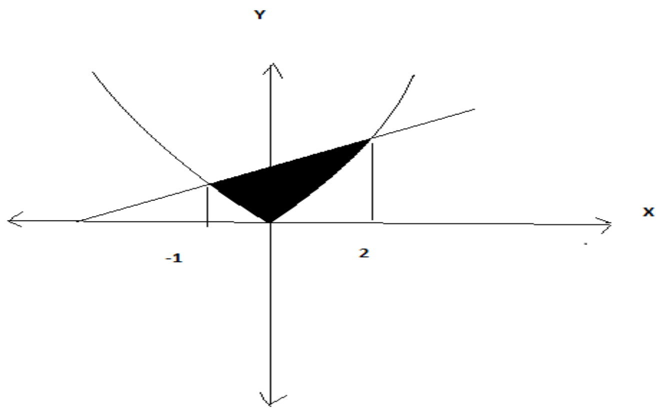
1</td></tr>
<tr><td>21</td><td>Any point on the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7} = \lambda$
$Q(3\lambda+1, 2\lambda, 7\lambda-1)$
P lies in the plane x + y – z = 8
$3\lambda + 1 + 2\lambda - 7\lambda + 1 = 8 \Rightarrow \lambda = -3$, hence Q(-8,-6,-22)
equation of line through P(1 , 3 , 2), Q(-8,-6,-22)
$\frac{x-x_1}{x-x_2} = \frac{y-y_1}{y-y_2} = \frac{z-z_1}{z-z_2} \Rightarrow \frac{x-1}{-9} = \frac{y-3}{-9} = \frac{z-2}{-24} = \frac{x-1}{3} = \frac{y-3}{3} = \frac{z-2}{8}$</td><td>½
1
1½

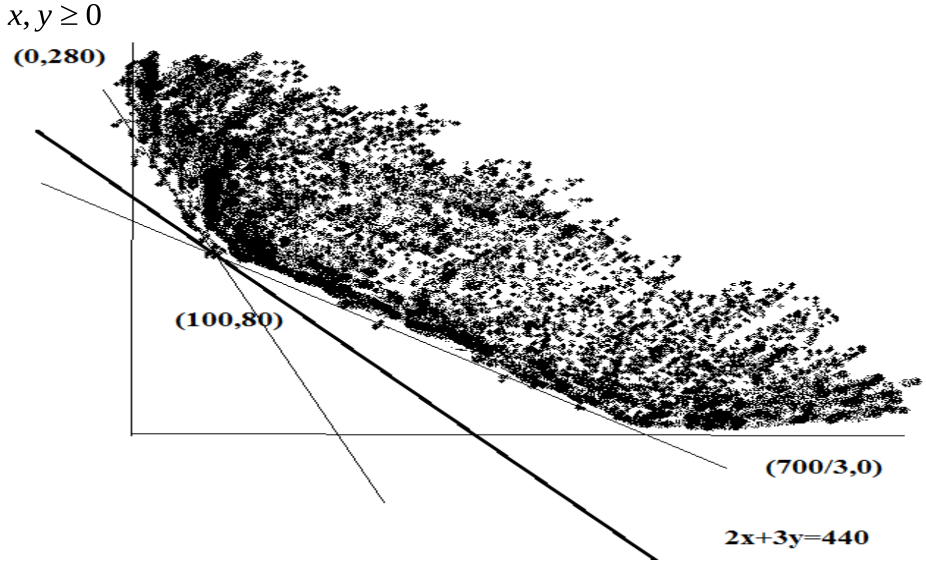
1</td></tr>
<tr><td>22</td><td>$(\tan^{-1} y - x)dy = (1+y^2)dx$
$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$
$\frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}$
$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$
$P = \frac{1}{1+y^2}, Q = \frac{\tan^{-1} y}{1+y^2}$
$\int P dy = \int \frac{1}{1+y^2} dy = \tan^{-1} y$
$e^{\int P dy} = e^{\tan^{-1} y}$
solution:
$xe^{\int P dy} = \int Q e^{\int P dy} dy + C$
$xe^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy$</td><td>½
½
1
½</td></tr>
</table>

	$RHS \quad \int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy$ substitute $\tan^{-1} y = t$ $\frac{1}{1+y^2} dy = dt$ $\int \frac{\tan^{-1} y}{1+y^2} e^{\tan^{-1} y} dy = \int t e^t dt - - - - - \text{int}\, egration\ by\ parts$ $u = t \Rightarrow du = dt,\ dv = e^t dt \Rightarrow \int dv = \int e^t dt \Rightarrow v = e^t$ $\int u dv = uv - \int v du = te^t - \int e^t dt = te^t - e^t \ = e^t (t-1) = e^{\tan^{-1} y}(\tan^{-1} y - 1)$ $\therefore xe^{\tan^{-1} y} \ = e^{\tan^{-1} y}(\tan^{-1} y - 1) + C$	1 ½
23	 Surface area = $\pi r l + \pi r^2 \Rightarrow l = \frac{s - \pi r^2}{\pi r}$ $V = \frac{1}{3}\pi r^2 h$ $\Rightarrow T = \frac{1}{9}\pi^2 r^4 h^2 \ = \frac{1}{9}\pi^2 r^4 [l^2 - r^2] = \frac{1}{9}\pi^2 r^4 \left[\left(\frac{s - \pi r^2}{\pi r}\right)^2 - r^2\right]$ $= \frac{1}{9}\pi^2 r^4 \left[\left(\frac{s^2 - 2s\pi r^2 + \pi^2 r^4}{\pi^2 r^2}\right) \ - r^2\right] = \frac{1}{9}\pi^2 r^4 \frac{[s^2 - 2s\pi r^2 + \pi^2 r^4 - \pi^2 r^4]}{\pi^2 r^2}$ $\Rightarrow T = \frac{1}{9}\pi^2 r^4 \left[\frac{s^2 - 2s\pi r^2}{\pi^2 r^2}\right] \Rightarrow T = \frac{1}{9} r^2 [s^2 - 2s\pi r^2] \Rightarrow T = \frac{1}{9}[s^2 r^2 - 2s\pi r^4]$ $\Rightarrow \frac{dT}{dr} = \frac{1}{9}[2s^2 r - 8s\pi r^3] = 0 \Rightarrow 2s^2 r - 8s\pi r^3 \Rightarrow S = 4\pi r^2$	½ ½ 2 1

	$l = \frac{s - \pi r^2}{\pi r} = \frac{4\pi r^2 - \pi r^2}{\pi r} = \frac{3\pi r^2}{\pi r} = 3r$	
	$\sin\alpha = \frac{r}{l} = \frac{r}{3r} \Rightarrow \alpha = \sin^{-1}\frac{1}{3}$	1
	$\frac{d^2T}{dr^2} = \frac{1}{9}\left[2s^2 - 24s\pi r^2\right] = \frac{1}{9}\left[2s^2 - 6s\left(4\pi r^2\right)\right] = \frac{1}{9}\left[2s^2 - 6s^2\right] = -\frac{4s^2}{9} < 0 \text{ max}$	1
	The volume of the cone is maximum when $\alpha = \sin^{-1}\frac{1}{3}$	
	(OR)	
	let length = breadth = x . height = h	
	volume $= 1024cm^3 \Rightarrow lbh = 1024 \Rightarrow x^2h = 1024 \Rightarrow h = \frac{1024}{x^2}$	1
	$C = 5(2lb) + 2.50[2h(l+b)] \Rightarrow C = 10x^2 + 5\left(\frac{1024}{x^2}\right)(2x \Rightarrow C \Rightarrow) 10x^2 + \frac{10240}{x}$	1
	$\frac{dC}{dx} = 20x + 10240\left(-\frac{1}{x^2}\right) = 0 \Rightarrow \frac{10240}{x^2} = 20x \Rightarrow x^3 = 512 \Rightarrow x = 8$	2
	$\frac{d^2C}{dx^2} = 20 - 10240\left(-\frac{2}{x^3}\right) = 20 + \frac{20480}{x^3} > 0 \quad \min imum$	1
	Least cost $= 10(64) + \left(\frac{10240}{8}\right) = 640 + 1280 =$ ₹ 1920	1
24	$I = \int_0^{\pi} \frac{x\sin x}{1+\sin x}dx$	
	$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin(\pi - x)}dx = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\sin x}dx$	
	$2I = \int_0^{\pi} \frac{\pi\sin x}{1+\sin x}dx$	1
	$2I = \pi\int_0^{\pi} \frac{\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} = \pi\int_0^{\pi} \frac{\sin x(1-\sin x)}{1-\sin^2 x}dx$	½
	$2I = \pi\int_0^{\pi} \frac{\sin x(1-\sin x)}{\cos^2 x}dx$	½
	$= \pi\int_0^{\pi} \frac{\sin x - \sin^2 x}{\cos^2 x}dx = \pi\int_0^{\pi}\left(\frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\right)dx$	1

	$=\pi\left[\int_0^{\pi}\left(\tan x\sec x-\tan^2 x)\right)dx\right]=\pi\left[\int_0^{\pi}\left(\tan x\sec x-(\sec^2 x-1)\right)dx\right]$ $2I=\pi[\sec x-\tan x+x]_0^{\pi}$ $=\pi[\sec\pi-\sec 0+\tan\pi-\tan 0+\pi]$ $2I=\pi[-1-1+\pi]=\pi[\pi-2]\Rightarrow I=\frac{\pi}{2}[\pi-2]$	1 2
	(OR) $I=\int_0^{\frac{\pi}{2}}\frac{x+\sin x}{1+\cos x}dx$ $I=\int_0^{\frac{\pi}{2}}\frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$ $I=\int_0^{\frac{\pi}{2}}\frac{x}{2\cos^2\frac{x}{2}}dx+\int_0^{\frac{\pi}{2}}\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$ $I=\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sec^2\frac{x}{2}dx+\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}dx$ $\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}dx-----Integration\ by\ \ parts$ $u=\tan\frac{x}{2}\Rightarrow du=\frac{1}{2}\sec^2\frac{x}{2}dx\ \ ,\ \ \int dv=\int dx\Rightarrow v=x$ $\int udv=uv-\int vdu$ $\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}dx=x\tan\frac{x}{2}-\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sec^2\frac{x}{2}dx$ $I=\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sec^2\frac{x}{2}dx+x\tan\frac{x}{2}-\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sec^2\frac{x}{2}dx=\left[x\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}=\frac{\pi}{2}\tan\frac{\pi}{4}=\frac{\pi}{2}$	 1 ½ 1 1 1½ 1

25	Point of intersection of x = 4y- 2 and $x^2 = 4y$	
	$x^2 = x+2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1,2$	1
		1
	Area of the shaded region $\int_{-1}^{2} \frac{x+2}{4} dx - \int_{-1}^{2} \frac{x^2}{4} dx$	1
	$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$	1
	$= \frac{1}{4}\left\{\left[2 - \frac{1}{2}\right] + 2[2+1] - \left[\frac{8}{3} + \frac{1}{3}\right]\right\}$ $= \frac{1}{4}\left[\frac{3}{2} + 6 - 3\right] = \frac{1}{4}\left[\frac{3}{2} + 3\right] = \frac{1}{4}\left[\frac{9}{2}\right] = \frac{9}{8}$ *square units*	2
26	$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$ $AX = B$	
	$\lvert A \rvert = 1(-6) - 2(-14) - 3(-15) = -6 + 28 + 45 = 67$	1
	$adjA = \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$	2
	$A^{-1} = \frac{adjA}{\lvert A \rvert} = \frac{1}{67}\begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$	½
	$X = A^{-1}B = \frac{1}{67}\begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}\begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}$	1
	$= \frac{1}{67}\begin{pmatrix} 201 \\ -134 \\ 67 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$	1
	$x = 3, y = -2, z = 1$	½

27	Let A denote the missing card as red, B missing card as black, C denote the first 13 cards drawn are red	½
	$P(A)=P(B)=\frac{26}{52}=\frac{1}{2}, P\left(C/A\right)=\frac{^{25}C_{13}}{^{51}C_{13}}, P\left(C/B\right)=\frac{^{26}C_{13}}{^{51}C_{13}}$	1½
	$P(B/C)=\frac{P(B)P\left(C/B\right)}{P(A)P\left(C/A\right)+P(B)P\left(C/B\right)}$	1
	$=\frac{\frac{1}{2}\times\frac{^{26}C_{13}}{^{51}C_{13}}}{\frac{1}{2}\times\frac{^{25}C_{13}}{^{51}C_{13}}+\frac{1}{2}\times\frac{^{26}C_{13}}{^{51}C_{13}}}=\frac{^{26}C_{13}}{^{25}C_{13}+^{26}C_{13}}=\frac{\frac{26!}{13!13!}}{\frac{25!}{13!12!}+\frac{26!}{13!13!}}=\frac{2}{3}$	2
	If the probability rate is high the player decides to go for it in the game of gambling.But the success rate will not be constant. By understanding the imbalance between success and failure rate one should not go for gambling game.	1
28	Let the no of kilograms of fertilizer of type I used be x kg and type II be y kg Minimize : z= 2x + 3y	1
	subject to $\frac{10}{100}x+\frac{5}{100}y\geq 14 \Rightarrow 10x+5y\geq 1400 \Rightarrow 2x+y\geq 280$ $\frac{6}{100}x+\frac{10}{100}y\geq 14 \Rightarrow 6x+10y\geq 1400 \Rightarrow 3x+5y\geq 700$	1
	$x, y \geq 0$  	1½

	| corner points | Z=2x+3y in rupees | | (100,80) | 440------ minimum | | (0,280) | 840 | | (700/3, 0) | 1400/3 | Hence 100kg of type I and 80 kg of type Ii have to be used to minimize the cost. Natural fertilizers- any two bird seed bags, seaweed, fish emulsion,seaweed,animals and human waste,earthworms,ashes,vegetable waste	1 ½ 1
29	Equation of plane passing through the points A (2 , 1 , 1) , B (1 , 2, 1) and C (1 , 1 , 2). $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$ $\begin{vmatrix} x-2 & y-1 & z-1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$ $\Rightarrow x+y+z-4=0$ Equation of line from P (1,1,1) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{1}=\lambda$ $Q(\lambda+1,\lambda+1,\lambda+1)$ Q lies in the plane x + y + z – 4 = 0 $\Rightarrow \lambda+1+\lambda+1+\lambda+1-4=0$ $\Rightarrow 3\lambda-1=0 \Rightarrow \lambda=\frac{1}{3}$ $\text{foot of perpendicular is } Q\left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right)$ distance from P(1,1,1,) on the plane x + y + z – 4 =0 $=\left\|\frac{1+1+1-4}{\sqrt{3}}\right\|=\frac{1}{\sqrt{3}}$	1 2 1 ½ ½ 1