

**KENDRIYA VIDYALAYA SANGATHAN, CHENNAI REGION**

**CLASS-XII- THIRD COMMON PRE-BOARD EXAMINATION-2012-2013**

**Subject: Mathematics**

**Time Allotted: 3hrs**

**Max.Marks-100**

**General Instructions:**

- (i) Total number of questions in the paper is 29.
- (ii) The question paper consists of 10 questions (Q.No.1 to 10) of 1 mark each, 12 questions (Q.No. 11 to 22) of 4 marks each and 7 questions (Q.No. 23 to 29) of 6 marks each.
- (iii) There is no overall choice. However internal choice has been provided in 4 questions of four marks and 2 questions of six marks. You have to attempt only one of the alternatives in all such questions.
- (iv) If you wish to answer any question, already answered for any reason, cancel the previous answer.
- (v) Use of calculator is not permitted.

SECTION A	
1	If A is a matrix of order $3 \times 3$ such that $ A  = -1,  B  = 2$ then find $ 3AB $
2	If $B = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ find $B^{4n}$ where $n \in N$
3	Evaluate : $\int e^x 7^x dx$
4	If $\cos^{-1}\left(\frac{a}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ , find 'a'
5	Evaluate : $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x} + \sqrt{x}} dx$
6	Find the value of p for which $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}, \vec{c} = p\hat{i} + (p-2)\hat{j} - \hat{k}$ are coplanar
7	If $\begin{pmatrix} 0 & x+3y \\ 12 & x \end{pmatrix}$ is a skew symmetric matrix, find the value of 'y'
8	Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

9	Let $\vec{a}$ and $\vec{b}$ be two vectors of same magnitude such that the angle between them is $60^\circ$ and $\vec{a} \cdot \vec{b} = 8$ , find the magnitude of $\vec{b}$
10	If $A = \{1,2,3\}$ find the number of relations on A containing ( 1, 2) and ( 1, 3) which are reflexive and symmetric but not transitive
<b>SECTION B</b>	
11	If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$ , prove that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ <b>(OR)</b> Differentiate $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1}x^2$
12	A bag contains 10 balls each marked with one digit from 0 to 9. If 2 balls are drawn at random, find the mean and variance of balls bearing a prime number.
13	The adjacent sides of a parallelogram represent the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ . Find the area of the parallelogram and the unit vectors parallel to its diagonals.
14	Find the particular solution of the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ given that $x = 0$ when $y = 1$ <b>(OR)</b> Solve : $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given $x = 0, y = 0$
15	Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$ . Show that $*$ is commutative and associative. find the identity element for $*$ on A, if any <b>(OR)</b> Show that the relation R on set $A = \{1,2,3,4,5\}$ given by $R = \{(a, b) :  a - b  \text{ is even}\}$ is an equivalence relation.
16	Evaluate $\int \frac{(5x-2)}{3x^2+2x+1} dx$ . <b>(OR)</b> Evaluate : $\int \frac{dx}{x(x^5 + 1)}$
17	Find the equations of the tangent and normal to the curve $x = \sin t$ and $y = \cos t$ at $t = \frac{\pi}{4}$

18	Using properties of determinants show that $xyz = -1$ , if $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ and $x \neq y \neq z$
19	Express $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ , $-\frac{\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.
20	Determine the value of 'a' so that f(x) is continuous at $x = 0$ $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & , x < 0 \\ a & , x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & , x > 0 \end{cases}$ <b>Mention any two factors that will affect the continuity of thoughts while writing the exams.</b>
21	Find the equation of the line passing the point (1, 3, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$ .
22	Solve the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)dx$
<b>SECTION C</b>	
23	Show that the semi vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$ <p style="text-align: center;"><b>(OR)</b></p> A metal box with a square base and vertical sides is to contain $1024\text{cm}^3$ of volume. The material for the top and bottom costs ₹ 5per $\text{cm}^2$ and that for the sides costs ₹ 2.50 per $\text{cm}^2$ . Find the least cost of the box.
24	Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$ <p style="text-align: center;"><b>(OR)</b></p> Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x + \sin x}{1 + \cos x} dx$
25	Using integration find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$
26	Solve the system of equations by matrix method : $x + 2y - 3z = -4$ ; $2x + 3y + 2z = 2$ ; $3x - 3y - 4z = 11$

27	<p>A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, what is the probability that the missing card is black?</p> <p><b>How the knowledge of probability does helps us to understand the imbalance between the success and failure rate in the game of gambling?</b></p>												
28	<p>A farmer has a supply of chemical fertilizer of type I and II as given in the table.</p> <table border="1" data-bbox="496 449 1200 726"> <thead> <tr> <th></th> <th colspan="2">amount in Kg</th> </tr> <tr> <th></th> <th>I</th> <th>II</th> </tr> </thead> <tbody> <tr> <td>Nitrogen</td> <td>10%</td> <td>5%</td> </tr> <tr> <td>Phosphoric acid</td> <td>6%</td> <td>10%</td> </tr> </tbody> </table> <p>After testing the soil conditions of a field, it is found that atleast 14 kg of nitrogen and 14 kg of phosphoric acid is required for a good crop. The fertilizer type I costs ₹ 2 per kg and type II costs ₹ 3 per kg. How many kilograms of each fertilizer should be used to meet the requirement and the cost be minimum.</p> <p><b>Suggest any two natural fertilizers that will be substituted for chemical fertilizers?</b></p>		amount in Kg			I	II	Nitrogen	10%	5%	Phosphoric acid	6%	10%
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29	<p>Find the distance of the point P ( 1 , 1 , 1 ) from the plane through the points A ( 2 , 1 , 1 ) , B ( 1 , 2 , 1 ) and C ( 1 , 1 , 2 ). Also find the foot of the perpendicular from P to this plane.</p>												